SERVICE LEARNING PROJECT – NUMERICALS

CLASS – 11th std

CHAPTER 1 :- NATURE OF PHYSICAL WORLD AND MEASUREMENT

<u>Question 1.</u>

In a submarine equipped with sonar, the time delay between the generation of a pulse and its echo after reflection from an enemy submarine is observed to be 80 sec. If the speed of sound in water is 1460 ms^{-1}. What is the distance of enemy submarine?

Answer:

Given:

Speed of sound in water = 1460 ms^{-1}

Time delay = 80s

Distance of enemy ship = ?

Solution:

Total distance covered = speed × time

= 1460 ms^{-1} × 80s = 116800 m

Time taken is for forward and backward path of sound waves.

$$\text{Distance of enemy ship} = \frac{\text{total distance covered}}{2} = \frac{116800}{2} m$$

= 58400 m (or) 58.4 km

<u>Question 2.</u>

Assuming that the frequency v of a vibrating string may depend upon

(i) applied force (F)

(ii) length (l)

(iii) mass per unit length (m), prove that $\nu \propto \frac{1}{l}\sqrt{\frac{F}{m}}$ using dimensional analysis.
[Related to JIPMER 2001]
Answer:
Given: The frequency v of a vibrating string depends
(i) applied force (F)
(ii) length (l)
(iii) mass per unit length (m)
Solution:

$$\nu \propto F^x l^y m^z \propto \nu = K F^x l^y m^z \quad \ldots(1)$$

Substitute the dimensional formulae of the above quantities

$$[M^0 L^0 T^{-1}] = [MLT^{-2}]^x [L] [ML^{-1}]^z$$

$$[M^0 L^0 T^{-1}] = [M^{x+z} L^{x+y-z} T^{-2x}]$$

Comparing the powers of M, L, T on both sides,
x + z = 0, x + y – z = 0, -2x = -1
Solving for x, y, z, we get

$\boxed{x = \frac{1}{2}}$ $\boxed{y = -1}$ $\boxed{z = -\frac{1}{2}}$

Substitute x, y, z values in equ(1)

$$\nu \propto F^{1/2} l^{-1} m^{-1/2} \quad \therefore \quad \nu \propto \frac{1}{l}\sqrt{\frac{F}{m}}$$

<u>Question 3.</u>
Jupiter is at a distance of 824.7 million km from the Earth. Its angular diameter is measured to be 35.72″. Calculate the diameter of Jupiter.

CHAPTER 2 : KINEMATICS

Question 1.

The resultant of two vectors A and B is perpendicular to vector A and its magnitude is equal to half of the magnitude of vector B. Then the angle between A and B is ……….

Question 2.

A object is thrown with initial speed 5 m s-1 with an angle of projection 30°. What is the height and range reached by the particle?

Solution :

Given,
Initial speed (u) = 5 ms^{-1}
Angle of projection θ = 30°
Solution:
Max height reached (h) = $\frac{u^2 \sin^2 \theta}{2g}$ = (25 x ¼) / (2 x 9.8) = 0.318 m

Range R = $\frac{u^2 \sin 2\theta}{g}$ = (25 x sin 60) / 9.8 = **2.21 m**

Question 3.

An object is executing uniform circular motion with an angular speed of 12 radian per second. At t = 0 the object starts at an angle q = 0 What is the angular displacement of the particle after 4 s?

Solution:

Angular speed = Angular displacement / time taken
Angular displacement = π12 x 4 = π12 = **60°**

CHAPTER 3 : LAWS OF MOTION

Question 1. A spider of mass 50 g is hanging on a string of a cob web. What is the tension in the string?

Solution: Given, m = 50 g

Tension in the string, T = mg

= 50 x 9.8 x 10^{-2} **= 0.49 N**

Question 2. A bob attached to the string oscillates back and forth. Resolve the forces acting on the bob in to components. What is the acceleration experience by the bob at an angle θ.

Solution: The gravitational force (mg) acting downward can be resolved into two components as mg cos θ and mg sin θ

T – tension exerted by the string.

Tangential force F_T = ma_T

T = mg sin θ

∴ Tangential acceleration a_T = g sin θ

Centripetal force F_c= ma_c = T – mg cos θ

a_c = (T – m g c o s θ) / m

<u>Question 3</u>. A long stick rests on the surface. A person standing 10 m away from the stick. With what minimum speed an object of mass 0.5 kg should he thrown so that it hits the stick. (Assume the coefficient of kinetic friction is 0.7).

CHAPTER 4 : WORK, ENERGY AND POWER

<u>Question 1</u>. A ball with a velocity of 5 ms−1 impinges at angle of 60˚ with the vertical on a smooth horizontal plane. If the coefficient of restitution is 0.5, find the velocity and direction after the impact.

Solution : Given:

Velocity of ball: 5 ms^{-1}

Angle of inclination with vertical: 60°

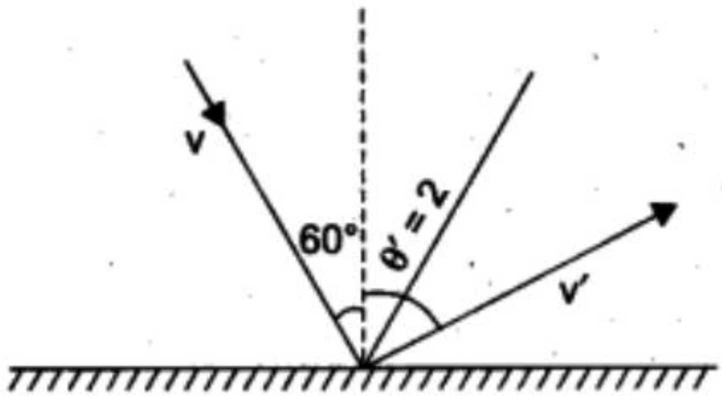
Coefficient of restitution = 0.5.

Note: Let the angle reflection is θ’ and the speed after collision is v’. The floor exerts a force on the ball along the normal during the collision. There is no force parallel to the surface. Thus, the parallel component of the velocity of the ball remains unchanged. This gives

v’ sin θ’ = v sin θ …… (i)

Vertical component with respect to floor = v’ cos θ’ (velocity of separation)

Velocity of approach = v cos θ



Coefficient of restitution $e = \dfrac{\text{velocity of separation}}{\text{velocity of approach}}$

$$e = \frac{v' \cos\theta'}{v \cos\theta} \quad \therefore \quad v' \cos\theta' = ev \cos\theta \quad \text{...(ii)}$$

From (i) and (ii)

(ii) $v'\sqrt{(1-\sin'^2\theta)} = ev\cos\theta$

$v'^2(1-\sin'^2\theta) = e^2v^2\cos^2\theta$

$v'^2 - v'^2\sin'^2\theta = e^2v^2\cos^2\theta$

$v'^2 = v^2\sin^2\theta + e^2v^2\cos^2\theta$ $\quad [v'\sin\theta' = v\sin\theta]$

$v' = \sqrt{v^2\sin^2\theta + e^2v^2\cos^2\theta}$

$v' = v\sqrt{\sin^2\theta + e^2\cos^2\theta}$

The speed after collision $v' = v\sqrt{\sin^2\theta + e^2\cos^2\theta}$

$$v' = 5\sqrt{\sin^2(60) + (0.5)^2\cos^2 60} = 5\sqrt{\frac{3}{4} + 0.25\times\frac{1}{4}}$$

$$= \frac{5}{2}\sqrt{3.25} = 2.5\times 1.8 = 4.5\ \text{ms}^{-1}$$

Angle of reflection $\theta' = \tan^{-1}\left(\frac{\tan\theta}{e}\right) = \tan^{-1}\left(\frac{\tan 60^\circ}{0.5}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{0.5}\right)$

$$= \tan^{-1}(3.464) = 73.9^\circ.$$

Question 2. Two different unknown masses A and B collide. A is initially at rest when B has a speed v. After collision B has a speed v/2 and moves at right angles to its original direction of motion. Find the direction in which A moves after collision.

Solution : Given u2 = v, u1 = 0

V2 = v/2, θ = 0

Component along x-axis

$m_1 v_1 \sin\theta = m_2 v$ …….. (1)

Component along y-axis

$m_1 v_1 \sin\theta = m_2\left(\frac{v}{2}\right) \ldots\ldots\ldots (2)$, from (1) and (2)

$$\tan\theta = \frac{\frac{v}{2}}{v} = \frac{1}{2}$$

$\theta = \tan^{-1}\frac{1}{2} = 26^0 33'$

Question 3. A bullet of mass 20 g strikes a pendulum of mass 5 kg. The centre of mass of pendulum rises a vertical distance of 10 cm. If the bullet gets embedded into the pendulum, calculate its initial speed.

CHAPTER 5 : MOTION OF SYSTEM OF PARTICLES AND RIGID BODIES

Question 1. A uniform disc of mass 100g has a diameter of 10 cm. Calculate the total energy of the disc when rolling along a horizontal table with a velocity of 20 cms-1.

Solution : Given, Mass of the disc = 100 g = 100 x 10-3 kg = 1/10 kg

Velocity of disc = 20 cm s-1 = 20 x 10-2 ms-1 = 0.2 ms-1

$$r = 5\text{ cm} = 5\times10^{-2}\text{ m},\quad \omega = \frac{v}{r} = \frac{20\times10^{-2}}{5\times10^{-2}} = 4$$

$$\text{Energy} = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(mV^2 + I\omega^2),\text{ where } I = \frac{1}{2}mr^2$$

$$= \frac{1}{2}\left[\frac{1}{10}\times0.2\times0.2 + \frac{1}{2}\times\frac{1}{10}\times25\times\frac{1}{10^4}\times16\right]$$

$$= \frac{1}{2}\left[\frac{4}{1000} + \frac{2}{1000}\right] = \frac{1}{2}\left[\frac{6}{1000}\right]$$

$$\text{Energy} = 3\times10^{-3}\text{ J}$$

Question 2. Two particles P and Q of mass 1kg and 3 kg respectively start moving towards each other from rest under mutual attraction. What is the velocity of their centre of mass?

Question 3. Find the moment of inertia of a hydrogen molecule about an axis passing through its centre of mass and perpendicular to the inter-atomic axis. Given: mass of hydrogen atom 1.7 × 10^{-27} kg and inter atomic distance is equal to 4 × 10^{-10} m.

Solution : Mass of hydrogen atom=1.7×10^{-27}kg

Total mass of two molecules =2×1.7×10^{-27}kg

distance between the two atoms =4×10^{-10}m

distance from the axis of rotation =4×10^{-10}m / 2 =2× 10^{-10}m

moment of inertia =I=mR^2

I= 2×1.7×10^{-27}kg × 2× 10^{-10}m x 2× 10^{-10}m

I=13.6×$\mathbf{10^{-47}}$ $\mathbf{kgm^2}$

CHAPTER 6 : GRAVITATION

Question 1. An unknown planet orbits the Sun with distance twice the semi major axis distance of the Earth's orbit. If the Earth's time period is T1 , what is the time period of this unknown planet?

Solution : By Kepler's 3rd law T2 ∝ a3
Time period of unknown planet = T2
Time period of Earth = T1
Distance of unknown planet from the Sun = a2
Distance of the Earth from the Sun = a1

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

$$T_2 = \left(\frac{a_2}{a_1}\right)^{\frac{3}{2}} T_1 \qquad a_2 = 2a,$$

$$T_2 = \left(\frac{2a_1}{a_1}\right)^{\frac{3}{2}} T_1 \Rightarrow \quad T_2 = 2\sqrt{2}\,T_1$$

Question 2. What is the gravitational potential energy of the Earth and Sun? The Earth to Sun distance is around 150 million km. The mass of the Earth is 5.9 × 10^{24} kg and mass of the Sun is 1.9 × 10^{30} kg.

Solution : Given, the mass of the Earth (m) = 5.98 × 10^{24} Kg and mass of the Sun (M) = 1.99 × 10^{30} Kg

The gravitational potential energy is given by:

U = -GMm/r

U = (6.673 x 10^{-11} x 5.9 x 10^{24} x1.9x10^{30})/(150x10^{9}) = **4987 x 10^{30}J**

Question 3. Suppose we go 200 km above and below the surface of the Earth, what are the g values at these two points? In which case, is the value of g small?

CHAPTER 7 : PROPERTIES OF MATTER

Question 1. A spherical soap bubble A of radius 2 cm is formed inside another bubble B of radius 4 cm. Show that the radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is lesser than radius of both soap bubbles A and B.

Solution : Excess pressure create with S.T of spherical surface of the liquid = ΔP = $2T/R$

T - surface tension

In case of soap bubbles,

The excess pressure of air inside them is double due to the presence of two interfaces are inside and one outside.

$\Delta P_b = 4T/R$

Excess pressure of air inside the bigger bubble

$\Delta P_b = 4T/4 = T$

Excess pressure of air inside the smaller bubble 4s 4T

$\Delta P_s = 4s/R = 4T/2 = 2T$

Air pressure difference between the smaller bubble and the atmosphere will be equal toll sum of excess pressure inside the bigger smaller bubbles.

Pressure different $\Delta P = \Delta P_b + \Delta P_s$

$= T + 2T = 3T$

Excess pressure inside a single soap bubble = $4T/R = 4T/4 = T$

Pressure difference of single soap bubble less than radius of both $T < 3T$

Question 2. A block of Ag of mass x kg hanging from a string is immersed in a liquid of relative density 0.72. If the relative density of Ag is 10 and tension in the string is 37.12 N then compute the mass of Ag block.

Solution : From the terminal velocity condition,

$F_G – U = F$

F = T, m = x

$$mg - mg\left(\frac{\rho_w}{\rho}\right) = T$$

$$mg\left[1 - \frac{0.72}{10}\right] = 37.12$$

$$9.8x\,[1 - 0.072) = 37.12$$

$$9.0944x = 37.12$$

$$\therefore \quad \boxed{x = 4\text{ kg}}$$

Question 3. A cylinder of length 1.5 m and diameter 4 cm is fixed at one end. A tangential force of 4 × 10^5 N is applied at the other end. If the rigidity modulus of the cylinder is 6 × 10^{10} N m^{-2}then, calculate the twist produced in the cylinder.

CHAPTER 8 : HEAT AND THERMODYNAMICS

Question 1. In the planet Mars, the average temperature is around -53°C and atmospheric pressure is 0.9 kPa. Calculate the number of moles of the molecules in unit volume in the planet Mars? Is this greater than that in earth?

Solution : Average temperature of Mars = $-53^0\mathrm{C}$

T mars = T1 = 273-53 = 220 K

Atmospheric pressure = P1 = 0.9kPa = 0.9 x 10^3Pa

PV = NKT …… (1)
Since V = 1, eq (1) becomes P = NKT

K is Boltzmann constant, N = P/KT = $0.9 \text{ X } 10^3$ / 1.381 X 220 X 10^{-23}
N = 2.964 X 10^{23}

μ = 2.964 X 10^{23} / 6.023 X10^{23} = 0.4921

$$\mu \text{ mass} = 0.49 \text{ mol}$$

$$N_A = \text{Avogrado's number}$$

No. of moles in unit volume in Mars = $\mu = \frac{N}{N_A}$

No. of moles in unit volume in Earth = 40 mol

Question 2. A man starts bicycling in the morning at a temperature around 25°C, he checked the pressure of tire which is equal to be 500 kPa. Afternoon he found that the absolute pressure in the tyre is increased to 520 kPa. By assuming the expansion of tyre is negligible, what is the temperature of tyre at afternoon?

Solution : For ideal gas equation of state PV = nRT

P1 = 500 kPa, T1 = 25°C = 25 + 273 = 298K, P2 = 520 kPa, T2 = ?

Expansion of tyre is negligible (V Constant)

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore T_2 = \left(\frac{P_2}{P_1}\right)T_1 = \frac{520}{500} \times 298 = \frac{154960}{500} = 309.92\text{K}$$

$$T_2 = 309.92 - 273 \; ; \; T_2 = 36.9°\text{C}$$

Question 3. A Carnot engine whose efficiency is 45% takes heat from a source maintained at a temperature of 327°C. To have an engine of efficiency 60% what must be the intake temperature for the same exhaust (sink) temperature?

CHAPTER 9 : KINETIC THEORY OF GASES

Question 1. If the rms speed of methane gas in the Jupiter's atmosphere is 471.8 m s^{-1}, show that the surface temperature of Jupiter is sub-zero.

Solution : RMS speed of methane gas (vrms) = 471.8 ms^{-1}

Molar mass of methane gas (M) = 16.04 g per mol

M = 16.04 x 10^{-3}kg/mol

Gas constant R = 8.31 $\text{mol}^{-1}\,\text{k}^{-1}$

$$v_{rms} = \sqrt{\frac{3\,RT}{M}}$$

$$(v_{rms})^2 = \frac{3RT}{M}$$

$$T = \frac{(v_{rms})^2 \times M}{3R} = \frac{(471.8)^2 \times 16.04 \times 10^{-3}}{3 \times 8.31}$$

$$= \frac{3.57 \times 10^6 \times 10^{-3}}{24.93} = 0.143 \times 10^3$$

$$T = 143\text{ K} - 273$$

$$T = -130°\text{C}$$

Question 2. Calculate the temperature at which the rms velocity of a gas triples its value at S.T.P. (standard temperature T1 = 273 K)

Solution : At STP temperature T1 = 273 K

RMS velocity of a gas (Vrms)1 = v

Net RMS velocity of a gas, at temperature (T2) (Vrms)2 = 3v

New temperature T2 = ?

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$$

$$T_2 = \left(\frac{(v_{rms})_2}{(v_{rms})_1}\right)^2 \times T_1 = \left(\frac{3v}{v}\right)^2 \times 273$$

$$T_2 = 9 \times 273 = 2457 \text{ K}$$

Question 3. Calculate the mean free path of air molecules at STP. The diameter of N2 and O2 is about 3 × 10^{-10}m.

CHAPTER 10 : OSCILLATIONS

Question 1. Consider a simple pendulum of length l = 0.9 m which is properly placed on a trolley rolling down on a inclined plane which is at θ = 45° with the horizontal. Assuming that the inclined plane is frictionless, calculate the time period of oscillation of the simple pendulum.

Solution : Length of the pendulum l = 0.9 m

Inclined angle θ = 45°

Time period of a simple pendulum T = 2π $\sqrt{l/g'}$

$$g' = g\cos\theta$$

$$T = 2\pi\sqrt{\frac{l}{g\cos\theta}} = 2 \times 3.14\sqrt{\frac{0.9}{9.8 \times \cos 45^\circ}} = 6.28 \times \sqrt{0.1298}$$

$$T = 2.263\text{s}$$

Question 2. A piece of wood of mass m is floating erect in a liquid whose density is ρ. If it is slightly pressed down and released then executes simple harmonic motion. Show that its time period of oscillation is T = then executes simple harmonic motion. Show that its time period of oscillation is T = $2\pi\sqrt{\frac{m}{Agp}}$

Solution : Spring factor of liquid = Aρg

Inertra factor of wood piece = m

Time period T = $2\pi\sqrt{\frac{\text{Inertia factor}}{\text{spring factor}}}$

T = $2\pi\sqrt{\frac{m}{A\rho g}}$

Question 3. Compute the time period for the system if the block of mass m is slightly displaced vertically down from its equilibrium position and then released. Assume that the pulley is light and smooth, strings and springs are light.

CHAPTER 11 : WAVES

Question 1. The speed of a wave in a certain medium is 900 m/s. If 3000 waves passes over a certain point of the medium in 2 minutes, then compute its wavelength?

Solution : Speed of the wave in medium v = 900 ms^{-1}

$$\text{Frequency}(n) = \frac{\text{Number of waves}}{\text{Time}} = \frac{3000}{2 \times 60} = 25s^{-1}$$

$$\text{Wavelength } \lambda = \frac{v}{n} = \frac{900}{25} \text{ ; } \lambda = 36 \text{ m}$$

Question 2. Consider a mixture of 2 mol of helium and 4 mol of oxygen. Compute the speed of sound in this gas mixture at 300 K.

Question 3. Consider two organ pipes of same length in which one organ pipe is closed and another organ pipe is open. If the fundamental frequency of closed pipe is 250 Hz. Calculate the fundamental frequency of the open pipe.

Solution : Fundamental frequency of closed organ pipe

$$f_c = \frac{v}{4l} = 250 \text{ Hz}$$

Fundamental frequency of open organ pipe

$$f_o = \frac{v}{2l} = ?$$

$$\frac{f_c}{f_o} = \frac{v}{4l} \times \frac{2l}{v} = \frac{1}{2}$$

$$f_o = 2f_c = 2 \times 250$$

$$\boxed{f_o = 500 \text{ Hz}}$$